# Comprehensive Course Syllabus

## BC Calculus 1

**Course Description:**

BC Calculus I is the first of a three-semester sequence designed to give a solid introduction to the study of Calculus. Students must have successfully completed MI-4, or its equivalent. The semester includes an intuitive approach to the rate of change of a function, limits, the definition of the derivative, and techniques for finding derivatives of various functions. Throughout the semester, students will also study a variety of applications of the derivative including many properties of functions, max/min problems, growth, and an introduction to differential equations.

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INSTRUCTOR(S):

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**Meeting Days, Time and Room(s)**
Monday, Tuesday, Thursday, Friday in A-155.

**Text(s) / Materials:**
Hughes-Hallett, D. , Gleason, A., McCallum, W. et al. (2009). *Calculus 5th Ed.*, John Wiley & Sons, Inc. ISBN 078-0470-13159-6.
Chapters 1 - 3, selected sections of Chapter 4, supplemental materials written by members of the IMSA Math Team.
Students are required to have a graphing calculator. TI-89 Titanium is recommended.
Students will also use *Mathematica*, and various internet resources via their laptops.

**Essential Content:**

**Understanding Derivatives**

- Slope of a tangent line
- Methods of approximation, including notion of step size
- $\Delta y = f'(x) \cdot \Delta x$

- Average vs. instantaneous rate of change
- Geometry of derivatives, including $f'$ and $f''$
- Graphically constructing derivatives and antiderivatives
- Interpretations in context

**Defining the Derivative and theory**
- Limits –
  - Intuitive understanding of a limit
  - Computation of basic limits
- Continuity of a function
- Definition of the derivative as a limit

**Computations of derivatives**
- Rules for differentiating elementary functions
- Product, quotient, and chain rules
- Implicit differentiation

**Applications of the derivative**
- Position, velocity, acceleration
- Optimization

**Differential Equations**
- Introduction to modeling quantities that change with time
- General and particular solutions

***IMSA Math Standards addressed:***

**A. Students studying mathematics at IMSA demonstrate a disposition and propensity to use mathematics, a variety of problem solving strategies, and creative thought to solve problems by:**

A.1 investigating and gaining insight into mathematical concepts by selecting and using a variety of traditional and creative problem solving strategies and methodologies. [SSL-I.B,III.B,III.C,IV.A]

A.3 interpreting, generalizing , and verifying the understanding gained in the problem solving process and extending it to new settings. [SSL-I.B,II.B,III.B,III.C,IV.A,IV.C; ]

A.5 demonstrating confidence, persistence, and reflective analysis of the effectiveness of an approach when attempting to solve a problem. [SSL-I.D,II.A,II.B,III.B,IV.A,IV.C]

**B. Students studying mathematics at IMSA reason logically in mathematical situations and understand the nature, role, and necessity of proof and counterexample in mathematical reasoning by:**

B.2 reasoning inductively and deductively. [SSL-I.B,III.C,IV.A]

B.3 making and testing conjectures, creating proofs, and identifying counterexamples. [SSL-III.C,IV.B]

B.4 enhancing inductive and deductive reasoning through the use of intuition, imagination, and other forms of reasoning. [SSL-III.C,IV.A]

B.5 analyzing and critiquing proofs created by themselves and others. [SSL-I.D,II.B,III.C,IV.A,IV.B,V.A]

B.6 understanding the role of logic in the development of mathematics and understanding the necessity of carefully proving assertions. [SSL-II.A,II.B,III.C,IV.D]

**C. Students studying mathematics at IMSA communicate clearly and accurately about mathematical relationships and results by:**

C.1 understanding mathematical information given in written, oral, symbolic, numeric, or graphic form and interpreting the relationship it represents. [SSL-IV.B]

C.2 accurately recording and effectively communicating using proper notation, vocabulary, and usage in a variety of modalities (written, oral, graphic, algebraic, etc.). [SSL-I.C,IV.B,V.A]

C.3 presenting mathematical work and results using the power of mathematical language effectively. [SSL-IV.B,V.A; IL-AoL-2,7.C; NCTM-8.2,8.4]

C.4 summarizing results in a form that is accurate, appropriate to the topic and level, and understandable to the intended audience. [SSL-I.C,IV.B,V.A]

**D. Students studying mathematics at IMSA demonstrate awareness of the inter-connectedness of mathematical thought in inter- and intra-disciplinary settings by:**

D.1 understanding that mathematics is a system of interconnected ideas. [SSL-III.B,III.C,IV.]

D.2 recognizing the commonalties among the components and processes of the sub-disciplines of mathematics. [SSL-I.B,III.B,III.C,IV.C]

**E. Students studying mathematics at IMSA understand and employ the power, economy, clarity, and elegance of mathematical representations by:**

E.1 recognizing that mathematical representations carry specific meanings and using mathematical notation correctly to enhance clarity and avoid ambiguity. [SSL-II.B,IV.B]

E.2 applying a variety of techniques to compare and manipulate mathematical representations. [SSL-I.A,III.B,IV.C]

E.3 recognizing the structure underlying a mathematical representation and utilizing this structure in analysis and problem solving. [SSL-III.B,IV.A,IV.C]

E.4 selecting an appropriate mathematical representation and demonstrating how it reflects the salient points of the situation it describes. [SSL-I.B,I.D,II.B,III.B,IV.A,IV.C]

**H. Students studying mathematics at IMSA understand and apply the concepts of change and invariance under change by:**

H.1 identifying, describing, and measuring various patterns of change. [SSL-I.C,IV.A,IV.B; IL-6.D,8.A]

H.2 applying limiting processes in graphical, numerical, and symbolic situations. [SSL-I.A,III.B]

H.3 applying concepts of change to problem situations using approximate or analytic methods as appropriate. [SSL-I.B,I.D,III.B]

**I. Students studying mathematics at IMSA understand and apply geometric relationships by:**

I.3 selecting and using appropriate geometric relationships, properties, formulas, tools, and units when working in a geometric context. [SSL-II.A,III.B,IV.A,IV.C; IL-7.A,9.A; NCTM-3.3,4.1,4.2]

I.6 solving problems involving coordinate (analytic) geometry. [SSL-I.A; IL-9.A,9.C; NCTM-3.2]

**L. Students studying mathematics at IMSA use technology to gain insight and obtain different perspectives on problems by:**

L.1 deciding whether to use technology, selecting an appropriate technology for a given situation, and understanding the limitation of the technology. [SSL-I.D,II.A,III.A]

L.2 using technology to facilitate doing, exploring, and understanding of mathematics. [SSL-II.A,III.A,IV.A]

L.3 judging the reasonableness of information and answers given by technology. [SSL-III.A,IV.A]

**SSLs and Outcomes:**

FA: Formally assessed, IA: Informally assessed

IA. Students are expected to demonstrate automaticity in skills, concepts, and processes that enable complex thought by

- completing daily homework assignments **FA, IA**
- completing regular assignments **FA**
- engaging in daily collaboration to complete or check work **IA**
- completing quizzes and tests **FA**

IB. Students are expected to construct questions, forge connections and deepen meaning by

- completing daily homework assignments **FA, IA**
- completing regular assignments **FA**
- engaging in daily collaboration to complete or check work **IA**
- completing quizzes and tests **FA**

IC. Students are expected to precisely observe phenomena and accurately record findings by

- regularly justifying conclusions and claims in all written work **FA**
- carefully supporting answers verbally with appropriate mathematical justification during in-class discussions **IA**
- engaging in daily collaboration to complete or check work **IA**

ID. Students are expected to evaluate the soundness and relevance of information and reasoning findings by

- regularly justifying conclusions and claims in all written work **FA**
- carefully supporting answers verbally with appropriate mathematical justification during in-class discussions **IA**
- engaging in daily collaboration to complete or check work **IA**

IIA. Students identify unexamined cultural, historical and personal assumptions and misconceptions that impede and skew inquiry by

- identifying weaknesses or misconceptions in related prior mathematical concepts **IA**
- discussing problems from multiple perspectives and opposing views to determine validity to various approaches **IA**
- engaging in daily collaboration to complete or check work **IA**

IIIA. Students use appropriate technologies as extensions of the mind by

- exploring mathematical ideas and problem solving using tools such as graphing calculators, Winplot, Mathematica, Excel, etc. **IA**
- making mathematical conjectures based on graphics and animations **IA**
- using web-based resources to clarify, verify, or explore ideas **IA**

IIIB. Students recognize, pursue, and explain substantive connections within and among areas of knowledge by

- applying calculus methods to familiar contexts, such as position, velocity and acceleration, and justifying conclusions **FA**
- solving problems that require similar means which involve new or less familiar application contexts and justifying conclusions **FA**

IVA. Students construct and support judgments based on evidence through

- ❖ constructing graphs of a function based on the graph of its rate of change, and vice versa, giving full written and/or verbal justification **FA**
- ❖ solving optimization problems, with full justification **FA**
- ❖ exploring and justifying solutions to differential equations **FA**

IVB. Students will be challenged to write and speak with economy, power, and elegance by

- ❖ supporting answers with written justification using precise mathematical notation and language **FA**
- ❖ making sound mathematical verbal arguments using precise language **IA**

VA. Students will identify, understand and accept the rights and responsibilities of belonging to a diverse community by

- ❖ actively participating in class discussions **IA**
- ❖ respecting each others' questions and responses, both in and out of class **IA**
- ❖ collaborating outside of class on Take Home and other assignments without infringing on each others' intellectual capital **IA**

VB. In order for students to make reasoned decisions which reflect ethical standards, and act in accordance with those decisions, students

- ❖ collaborate outside of class on assignments without infringing on each others' intellectual capital **IA**
- ❖ produce their own work on formal assessments **FA**

**Instructional Design and Approach:**

Class time will regularly involve collaboration, investigation, and communication. Collaboration encourages discussion between students who are enthusiastic about learning; they share ideas amongst themselves from multiple perspectives, which in turn encourages students to construct their own understanding. Investigation includes the frequent use of technology to help forge connections between different representations, promoting a deeper understanding of concepts. Students are encouraged to think independently and draw upon experiences from other classes as a natural part of the investigative process. Students are expected to delve deeply into content, forming rigorous and broad connections within and among concepts. Communication is the tie that binds collaboration and investigation. It allows students to work together and share ideas, allows the teacher to assess and to push students further, and it helps students to monitor their own understanding.

**Student Expectations:**

In-class work – This will take the form of worksheets or problems assigned from your text or other sources. This work will often form the basis for explorations, class discussions, and homework. It needs to be completed in a timely manner, usually by the beginning of the next class period.

Homework – You can expect homework assignments most every evening from the book or from Webwork. These will include problems and reading assignments from the textbook and need to be completed by the beginning of the next class period,

unless otherwise indicated. At various times during the semester, we will collect individual homework assignments or problems and return them with corrections. Your homework should be neat and written out in a clear manner so that we are able to follow the reasoning that leads to your solution. In addition, 4-6 times during the semester, you will be given longer homework assignments known as "Take-Homes" (not to be mistaken for MI Problem Sets). You may either write your homework neatly, type your homework in the TeX typesetting language, or Mathematica. If a collected homework assignment is turned in late without prior approval of the instructor, then 10% will be deducted per day that the assignment is late.

Projects – You will have at least one project this semester. You will be assessed on the quality of the mathematics as well as the quality of the presentation, which may be written and/or oral. Further details will be provided when necessary.

Quizzes and Tests – These will be given throughout the quarter. All work must be shown.

**Assessment Practices, Procedures, and Processes:**

Grading policy:
Quarter grade:75% from Tests and Quizzes
25% HW (5% Webwork, 5% Worksheets, 10% Take-Homes, 5% Projects)

Semester grade: 80% from cumulative semester work
20% from the semester final exam

Semester grades will be averaged using the following weightings:

| | |
|---|---|
| Each Quarterly Grade | 40% |
| Semester Exam | 20% |

Grading scale:
A - 90% or above
B - 80% or above but less than 90%
C - 70% or above but less than 80%
D - less than 70%

**Sequence of Topics and Activities**

Chapters 1 - 3, first two sections of Chapter 4, supplemental materials written by members of the IMSA Math Team.

| Days | Content |
|---|---|
| 1 – 10 | Sections 1.1 – 1.6: Review – mostly.<br>Worksheets: Startup 1, 2, 3 (review) |
| 5 – 10 (overlapping) | Worksheets: Rate of Change<br>Basic relationships between position and velocity graphs, moving in both directions. Introduce Euler's Method (where $y' = f(x)$ only) to help connect the graphs of $y$ and $y'$ and to introduce the importance of step size and approximations. |
| 11 – 17 | Sections 1.7 – 1.8: Intro to continuity and limits. Intermediate Value Theorem. Worksheets for extra work on limits. Use graphs, algebra, |

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|---|---|
| | tables, and intuition for limits as $x \to a$, as $x \to \infty$, and limits that approach $\infty$. Asymptotes. One-sided limits and continuity for various functions. Introduction to $\delta$ - $\varepsilon$ definition. |
| 18 – 21 | Sections 2.1 – 2.2: Measuring speed and definition of a derivative. Average rate of change vs. instantaneous rate of change. Local linearity and tangent lines. |
| 22 – 31 | Sections 2.3 – 2.6: Meaning of the derivative, derivative function, second derivative and concavity. Intro to geometry of derivatives with $f$, $f'$, and $f''$ with extra worksheets. If differentiable, then continuous. |
| 32 – 40 | Section 3.1 – 3.4: Rules for power functions, exponential functions (worksheet). Product, quotient, and chain rules. |
| 41 – 43 | Sections 3.5: Trig limits (with worksheet) and derivatives of trig functions. |
| 44 – 53 | Sections 3.6 – 3.7, Derivatives of inverse functions, including the natural log and inverse trig functions. Implicitly defined functions and derivatives. Intro to Related Rates. |
| 54 – 57 | Sections 3.9 – 3.10: Linear approximations, Mean Value Theorem, more theorems. |
| 58 – 63 | Section 4.1 – 4.2: More on geometry of derivatives. Intro to optimization. Extreme Value Theorem. |
| 64 – 66 | Semester Review |